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“Vienna VLBI2010 PPP Simulator”

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Abstract

Though a theoretical approach in terms of VLBI, the Precise Point Positioning (PPP) Simulator has proven to be a very helpful tool. It is based on MATLAB script files and computes either a classical least squares (CLS) or a Kalman filter (KF) solution. In this memo, the advantages of the PPP simulator will be pointed out. The two approaches, CLS and KF, will be described briefly. Finally, PPP CLS and PPP KF results will be compared to results of the simulator version of OCCAM Kalman filter.

1. Introduction

In addition to the VLBI2010 Monte Carlo simulators based on the VLBI analysis software packages OCCAM (Wresnik et al., 2007a) and Calc/Solve (MacMillan, 2007), a VLBI2010 precise point positioning (PPP) Simulator was developed at the Institute of Geodesy and Geophysics, Vienna University of Technology. Though a theoretical approach in terms of VLBI, it has several advantages that make it a powerful tool for VLBI2010 simulation studies:

- It is based on MATLAB script files – the source code is compact and clear making it easy to implement modifications.
- Since it is evaluating single stations only, it is time saving – many different strategies can be tested in fairly short time.
- Due to its simplicity it supports the basic understanding of the main stochastic error sources, tropospheric wet delay and clock.

As for the OCCAM and Calc/Solve simulations, time series of artificial group delay observables $delay_{group}$ are generated, consisting of tropospheric equivalent zenith wet delays zwd (provided by a generator based on a turbulence model), stochastic variations of station clock (clk) and white noise (wn) to account for the thermal noise of the receiving system. The equivalent zenith wet delays need to be mapped with a wet mapping function mfw (which is considered to be free of error) to the elevations el of the observations:

$$delay_{group} = zwd \cdot mfw(el) + clk + wn . \quad (1)$$

The equivalent zenith wet delays are generated with a turbulence simulator following the approach proposed by Nilsson et al. (2007) which is based on the Treuhaf and Lanyi turbulence model (1987). The parameters driving the turbulence, refractive index structure constant C_n , effective height of wet troposphere H , and wind velocity vector \mathbf{v} , are provided by Tobias Nilsson from Onsala Space Observatory, Sweden (see Nilsson 2008 for more details on how the parameters are derived). The stochastic variations of station clock are computed as sum of random walk and integrated random walk (Böhm et al., 2007), typically with power spectral densities corresponding to Allan standard deviations (ASD) of $1e-14$ @ 50 min or $2e-15$ @ 15 min. For OCCAM and Calc/Solve simulations, a white noise of 4 ps

per baseline, which is the nominal goal of VLBI2010, is added. For PPP this corresponds to a white noise of $4/\sqrt{2}$ ps per station.

2. The VLBI2010 PPP Simulator

The fake delay observables (1) form the input to the PPP Simulator. The PPP itself is performed either with the classical least squares (CLS) method or a Kalman filter (KF) which will be described in more detail below. The parameters to be estimated are troposphere parameters, clock parameters and station position residuals. Goal is to regain the single contributions of troposphere and clock to the cumulative delay as accurately as possible. The separation of these parameters is possible due to their distinct dependencies on elevation angle. This procedure is repeated 25 times to obtain a sample of output parameters that can be analyzed statistically. The results are position repeatabilities, and rms values of zenith wet delay and clock post fit residuals.

2.1 PPP with classical least squares

The classical least squares method, also referred to as Gauss-Markov method, is widely known and shall not be discussed in detail here. Troposphere and station clock are treated as stochastic parameters. The tropospheric slant delay swd is modeled with a zenith wet delay zwd and superimposed gradients as proposed in the IERS Conventions 2003 (McCarthy and Petit, 2004):

$$swd = zwd \cdot mfw(el) + \cot(el) \cdot mfw(el) \cdot [grd_N \cdot \cos(az) + grd_E \cdot \sin(az)]. \quad (2)$$

In (2) mfw denotes wet mapping function, el elevation angle, grd_N and grd_E north and east gradient respectively, and az azimuth. The parameters to be estimated thus are zwd , grd_N and grd_E , all of which are parameterized as piecewise linear functions. One offset is estimated at the first time epoch and then one rate per estimation interval. The rates are constrained to zero by introducing pseudo observations. Station clock is modeled with a second order polynomial and superimposed piecewise linear functions. As for troposphere parameters, the clock rates are constrained to zero.

The components of station position are treated as deterministic parameters and are estimated once per 24 hour session.

2.2 PPP with Kalman filter

The KF version of the PPP Simulator is based on a covariance propagating Kalman filter as described thoroughly by e.g. Gelb (1974). Figure 1 shows a flow chart of the Kalman filter algorithm. For initialization of the filter, a priori values \mathbf{x}_0 with variances (and eventually covariances \mathbf{C}_0) are required for the parameters to be estimated. A mathematical formulation of the models used to describe reality is contained in the state transition matrix \mathbf{A} . Information about how well these models represent reality needs to be given in form of the system noise matrix \mathbf{Q} . With this information, the filter can be initialized as shown in Figure 1.

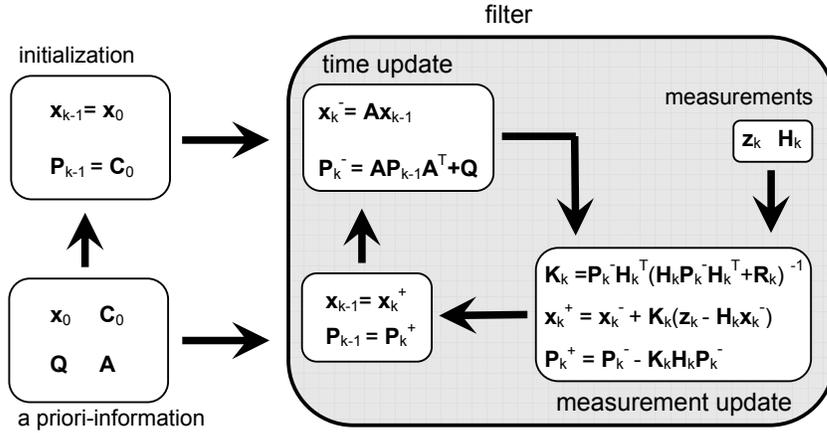


Figure 1: Flow chart of a covariance based Kalman filter

The filter algorithm itself starts after the initialization. The equations can be divided into two groups: the time update equations (also referred to as Predictor-Equations) and the measurement update equations (also referred to as Corrector-Equations).

With the system state vector \mathbf{x}_{k-1} at time epoch t_{k-1} , the corresponding covariance matrix \mathbf{P}_{k-1} , and the matrices \mathbf{A} and \mathbf{Q} , the system state vector \mathbf{x}_k^- and its covariance matrix \mathbf{P}_k^- at time epoch t_k can be predicted. Then, the measurement vector \mathbf{z}_k at time epoch t_k and the design matrix \mathbf{H}_k , that is linking \mathbf{z}_k and the system state vector, are used to correct the predicted system state vector \mathbf{x}_k^- . The corrected system state vector \mathbf{x}_k^+ and its corrected covariance matrix \mathbf{P}_k^+ are the final estimates of the filter at time epoch t_k . These results are used to predict the system state vector at the next time epoch and this loop is repeated until all measurements have been included.

In the filter estimate at time epoch t_n , only the measurements between the first time epoch and time epoch t_n are included. For estimation of stochastic parameters, all available information should be included. Thus, Herring et al (1990) proposed to use smoothing instead of filtering. The principle of smoothing is shown in Figure 2. A smoother is a combination of two Kalman filters running in opposite directions. The forward running filter (FRF) starts at the first time epoch t_{first} , the backward running filter (BRF) starts at the last time epoch t_{last} . At time epoch t_n both filters together contain all available information (see Figure 2).

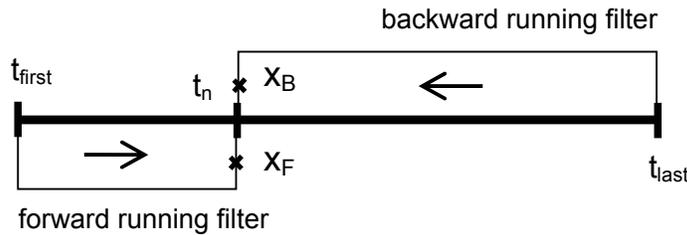


Figure 2: Principle of smoothing (Gelb 1974)

Be \mathbf{x}_F the FRF estimate of the system state vector and \mathbf{x}_B the BRF estimate of the system state vector at time epoch t_n . Then, the smoothed estimate of the system state vector \mathbf{x}_S can be computed as

$$\mathbf{x}_S = \mathbf{x}_F + \bar{\mathbf{P}}(\mathbf{x}_B - \mathbf{x}_F)^{-1}, \quad (3)$$

where $\bar{\mathbf{P}}$ is a weighting matrix given by

$$\bar{\mathbf{P}} = \mathbf{P}_F(\mathbf{P}_F + \mathbf{P}_B)^{-1} \quad (4)$$

with \mathbf{P}_F being the covariance matrix of the FRF estimate and \mathbf{P}_B being the covariance matrix of the BRF estimate. The covariance matrix of the smoothed system state vector can finally be computed by

$$\mathbf{P}_S = (\mathbf{P}_F^{-1} + \mathbf{P}_B^{-1})^{-1}. \quad (5)$$

Station coordinate residuals are treated as deterministic parameters. Their final estimate is the final estimate of the FRF. Troposphere and clock are treated as stochastic parameters. Their final estimates are thus taken from the smoothed solution at each time epoch. Zenith wet delay is modelled as random walk with a variance rate of typically $0.7 \text{ ps}^2/\text{s}$. As for the CLS, the zenith wet delay is superimposed with gradients which are also estimated as random walks with a variance rate of $0.5 \text{ ps}^2/\text{s}$. Station clock is composed of a rate, treated as deterministic parameter, and an offset, treated as stochastic parameter and estimated as random walk with a variance rate of typically $1 \text{ ps}^2/\text{s}$. These variance rates are used for all stations and fit the variance rates used for the OCCAM VLBI2010 simulations (Wresnik et al., 2007b).

3. Validation and Results

For validation of the PPP Simulator and to assess how close the PPP results are to what can be expected from real VLBI data, comparisons to OCCAM Kalman filter results were carried out. The comparison was carried out for four 16 station VLBI2010 test schedules: st16uni_15_3_230, st16uni_30_6_230, st16_uni_45_9_230 and st16uni_60_12_230. These schedules were generated by Toni Searle from NRC, Canada and are characterized by uniform sky coverage over time intervals of 3, 6, 9, and 12 minutes respectively. For more information about these schedules see Petrachenko et al (2008).

For all 16 stations and each schedule, a PPP was performed using the same time series of turbulent equivalent zenith wet delay and station clock as were used for the OCCAM simulations. For each station and each of the 25 days of data, a 3D position error was computed. To obtain a single value to compare, an rms of 3D position error was computed over the 25 days for each station.

These 3D rms error values are plotted in Figure 3 for the four schedules. OCCAM results are plotted in blue, PPP results in red. From this plot it can be deduced that the solutions of the two simulators fit pretty well. In general, the PPP solution is a little better, what had to be expected since with PPP it is not necessary to separate the tropospheres of two stations forming a baseline.

Figure 4 shows rms of zenith wet delay and clock post fit residuals for the forward running filter, the backward running filter and the smoothed solution. Although the results of the backward running filter are worse than the results of the forward running filter, the smoothed solution is best. The filter recognizes that the backward solution is worse than the forward solution and assigns less weight to the backward solution when performing the smoothing.

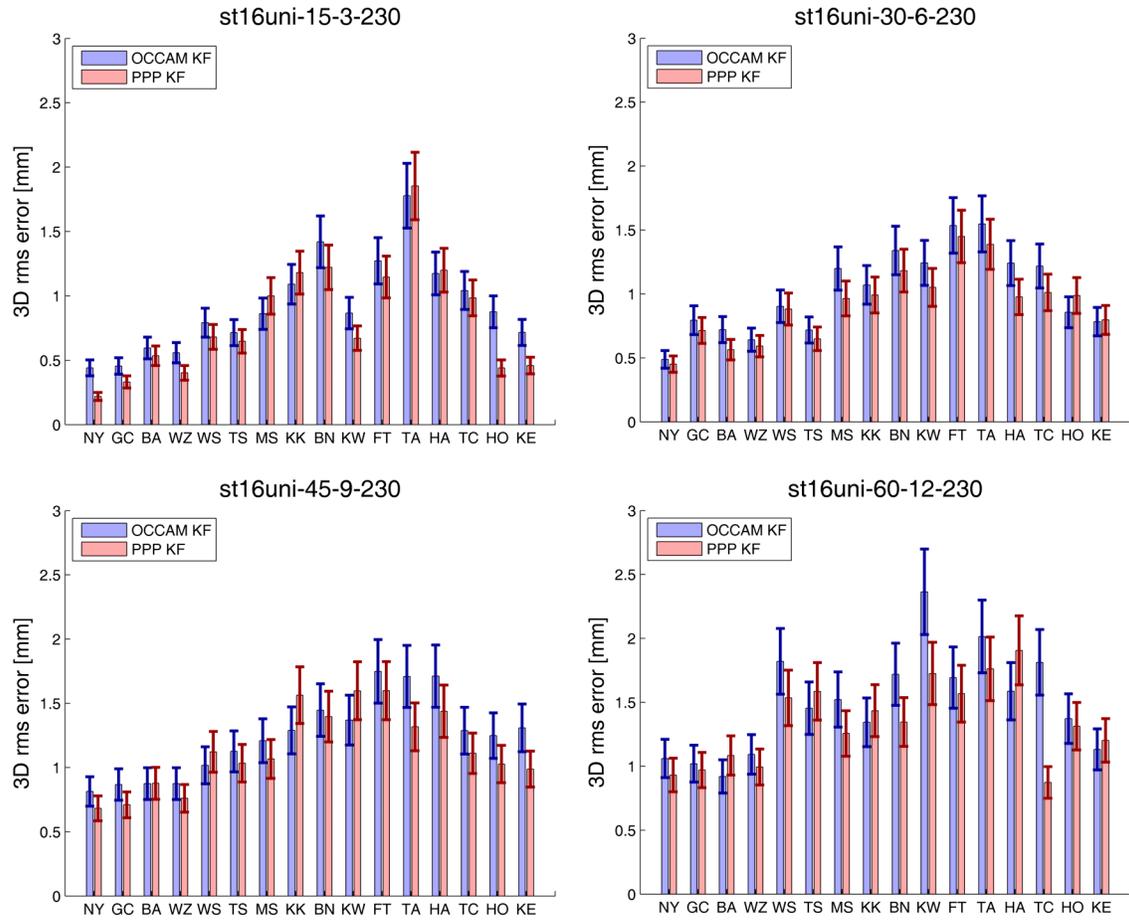


Figure 3: rms of 3D position error in mm for four VLBI2010 test schedules. OCCAM Kalman filter results can be seen in blue, PPP Kalman filter results in red. The error bars show 1 sigma of the scatter ($\text{rms}/\sqrt{2n}$) where $n = 25$). ASD $1e-14 @ 50$ min, $w_n = 4/\sqrt{2}$ ps for PPP and 4 ps for OCCAM. Variance rates: $0.7 \text{ ps}^2/\text{s}$ (zwd), $0.5 \text{ ps}^2/\text{s}$ (grd), and $1 \text{ ps}^2/\text{s}$ (clk offset).

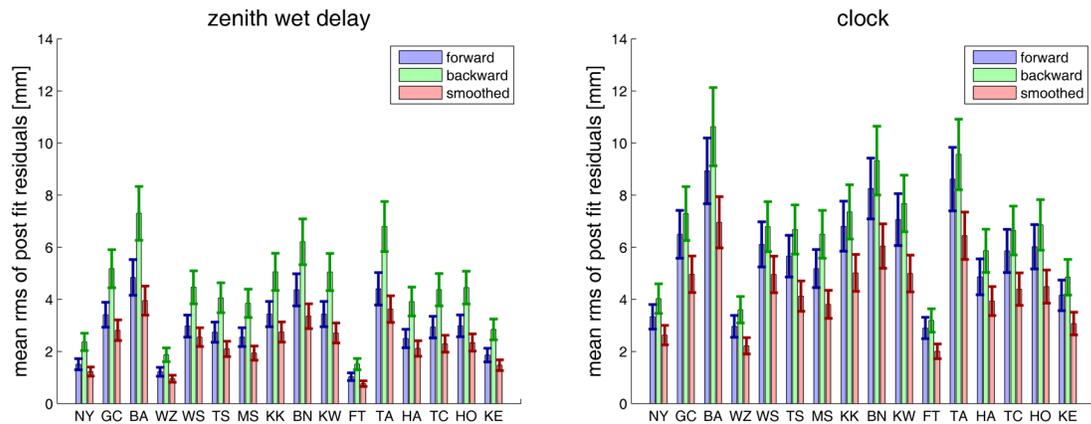


Figure 4: mean rms of post fit residuals in mm for forward solution (blue), backward solution (green), and smoothed solution (red). Schedule: st16uni_60_12_230.skd, ASD $1e-14 @ 50$ min, $w_n = 4/\sqrt{2}$ ps. Variance rates: $0.7 \text{ ps}^2/\text{s}$ (zwd), $0.5 \text{ ps}^2/\text{s}$ (grd), $1 \text{ ps}^2/\text{s}$ (clk offset). The errorbars show 1 sigma of the scatter ($\text{rms}/\sqrt{2n}$) where $n = 25$).

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